Int J Theor Phys (2010) 49: 2320–2333
DOI 10.1007/s10773-010-0419-x

Unification Theory of Different Causal Algebras and Its Applications to Theoretical Physics

Yong-Chang Huang · Changyu Huang · Bin He · Shi-Lin Yang

Received: 25 February 2010 / Accepted: 22 June 2010 / Published online: 13 July 2010

Abstract This paper gives a generalization of group theory, i.e. a unification theory of different causal algebras, and its applications to theoretical physics. We propose left and right causal algebras, left and right causal decomposition algebras, causal algebra and causal decomposition algebras in terms of quantitative causal principle. The causal algebraic system of containing left (or right) identity I_{jL} (or I_{jR}) is called as the left (or right) causal algebra, and associative law is deduced. Furthermore the applications of the new algebraic systems are given in theoretical physics, specially in the reactions of containing supersymmetric particles, we generally obtain the invariance of supersymmetric parity of multiplying property. In the reactions of particles of high energy, there may be no identity, but there are special inverse elements, which make that the relative algebra be not group, however, the causal algebra given in this paper is just a tool of severely and directly describing the real reactions of particle physics. And it is deduced that the causal decomposition algebra is equivalent to group.

Keywords Group theory · Particle interaction · Causal principle · Supersymmetry · Algebra · Semigroup

1 Introduction

In investigating physical phenomena, up to now, people have been finding different symmetries, many revolutions in physics are relative to different symmetries. For example, the

Y.-C. Huang (✉)
Institute of Theoretical Physics, Beijing University of Technology, Beijing, 100124, China
e-mail: ychuang@bjut.edu.cn

C. Huang
College of Physics, Peking University, Beijing, 100871, China

B. He · S.-L. Yang
Institute of Applied Mathematics, Beijing University of Technology, Beijing, 100124, China

establishment of four kinds of basic gauge interactions and discovery of fundament particles are relative to the investigations of symmetric property [1–3], these symmetries can be described by means of group [4].

Shifman and Turbiner studied Energy-reflection symmetry of Lie-algebraic problems [5], Ref. [6] systematically investigated current algebra and the Ademollo-Gatto theorem in spin-flavor symmetry of heavy quarks, Quantum algebra as the dynamical symmetry of the deformed Jaynes-Cummings model is in detail explored [7], and Anderson systematically studied a formal Lie-algebra of approaching to symmetry breaking in an attempt to reduce the arbitrariness of Lagrangian (Hamiltonian) models which include several free parameters and/or *ad hoc* symmetry groups [8].

In physical viewpoint, why do the material structure and motion have so many symmetric properties? There must exist general reasons of governing different physical phenomena. In fact, causal principle should be satisfied in expressing physical laws. Classical systems have deterministic causal evolution laws; in quantum mechanics there must be statistical causal evolution laws of some mathematical formulations. Furthermore, both in special theory of relativity and in general theory of relativity, the causal principle must be satisfied. There are too many laws satisfying causal principles to give their all examples in physics, therefore investigating whole physics may know that there are the laws: Any science in physics must satisfy some kinds of causal principles, and the causal relations must be quantitative in some mathematical formulations, or there must be contradictory. And it is well known that in physics the admitted physical laws not satisfying causal principle have not been found. In terms of the condition that causal principle is quantitative, i.e., quantitative causal principle (QCP), unified expressions of all differential and integral variational principles are given [9, 10]. Utilizing the no-loss-no-gain homeomorphic map transformation satisfying QCP, Ref. [11] gains the strain tensor formulas in condensed theory, it still satisfies QCP to investigate the solution of the cosmic quantal evaluative Wheeler-De Witt equation and the generalization of classical statistical mechanics to quantum mechanics [12, 13].

In fact, the symmetric properties in physics and mathematics are just the invariant properties under some kinds of operations of systems [14]. Using QCP we find that the four axioms of group theory are not independent, the unified descriptions of axioms of group are given in this paper.

It is well known, in the past, that the set satisfying the four mutual independent axioms is defined as group, yet we find that the four so-called independent axioms can be deduced from QCP, i.e., we make the four independent axioms turn into the four theorems. Furthermore, we also find that the four axioms (which now can be called theorems) are not mutual independent, namely, only three of the four axioms (or called theorem) are mutual independent.

As is known to all, a practical theory system is normally formed according to the rigorous mathematical logic deduction system. First of all, induce several basic principles (or say axioms); second, according to those principles deduce some theorems and lemmas; then, from these theorems and lemmas educe the whole logic deduction system; at last, apply the whole logic system into practice to achieve the practical theory system. This paper is based on the procession above to form a practical theory system. First of all, according to the rigorous mathematical logic deduction system, induce a principle; second, from the principle, deduce some theorems and lemmas; then, on the basis of these theorems and lemmas, deduce the whole logic deduction system; finally, apply the whole logic system into physics practice to achieve the practical theory system.

The plan of the paper is: Sect. 2 gives Left causal algebra, Sect. 3 gives right causal algebra, Sect. 4 proposes left and right causal decomposition algebras, Sect. 5 shows up causal algebra, Sect. 6 represents causal decomposition algebra, Sect. 7 discusses Relations between causal decomposition algebra and group, Sect. 8 is application and discussion, the last section is summary and conclusion.

2 Left Causal Algebra

In a general physical system, the quantitative actions (cause) of some quantities must lead to the relative equal effects (result) of the other quantities so that the system keeps a kind of invariant property [9, 10], therefore we have [9, 10, 12, 13].

Quantitative Causal Principle (QCP) In a general physical system, how much loses (cause), there must be, how much gains (result), or quantitative actions (cause) of some quantities must lead to the relative equal effects (result) of the other quantities so that the system keeps a kind of invariant property.

Then, the principle may be concretely written as

$$D(S)=CS, \tag{2.1}$$

Equation (2.1)'s physical meanings are that the real physical result produced by any operator function set D acting on S must lead to appearance of set C acting on S so that $D(S)$ ($D(S)$ represents that the operator function D takes a general operator function $D(S)$ relative to S, in which a simple case is $D(S)=DS$) is equal to CS, where D and C may be different sets, the whole process satisfies the QCP so that (2.1)'s left hand side equates right hand side, i.e., keeping no-loss-no-gain, so that the system keeps a kind of invariant property [9–11]. It can be seen that gauge fields of physical fundamental interactions are just connections of principal bundle [15], the connections are obtained by (2.1) when D is a connection operator, S is a section of the bundle, C is a connection ω [16], and different basic interactions in physics are mediated by corresponding gauge fields, thus the QCP is essential. In fact, the Noether theorem and conservation law of energy transformation are two of a lot of applied examples of quantitative causal principle with the no-loss-no-gain characteristic.

For group, ring, field and module, their different units have different influences on them. For the research of these algebraic structures, people are still in quest, and acquired many conclusions, e.g., for universal algebra [17]. For the research of Hopf algebra, quantum group etc, the algebras presented in Refs. [18, 19] are also the algebras of satisfying QCP. This paper studies from a new aspect, in order to make above-mentioned different algebraic systems have the corresponding development. We now carry on the general thorough research below.

In QCP's expression (2.1), when $D=S$ is a non-empty set, C identity, then $DS=S$, i.e., $DS=S$ is just the closure derived from (2.1). Accordingly, for any $(\forall)A_i, A_j, A_m \in S$, there is a general condition

$$\text{(i)}\quad A_i * A_j = A_m, \quad i,j,m\in Z \quad \text{(closure)}. \tag{2.2}$$

Equation (2.2) shows that A_j is acted from the left by A_i and transferred to A_m. According to closure (2.2), one has

$$A_j * A_k = A_l, \quad j,k,l\in Z. \tag{2.3}$$

We generally define that there exists a general inverse element A_m, namely (ii): $\forall A_k$ is acted from the left by A_j, then A_m is acted from the left by A_i, again $(A_j * A_k)$ is acted from the left by $(A_i * A_m)$, therefore, we obtain $D(S) = \{(A_i * A_m) * (A_j * A_k)\}$, iff $C = S$, namely, $CS = \{A_i * A_k\}$, we finally achieve: $\forall A_i, A_k \in S$, there is $A_j, A_m \in S$, there exists ($\exists$) the reciprocal eliminable condition

$$(A_i * A_m) * (A_j * A_k) = A_i * A_k, \tag{2.4}$$

that is, in terms of (2.1), we generally obtain a general inverse element A_m satisfying (2.4).

In (2.4), taking A_j as A_m, then the relative original A_m needs to remark as $(A_m)^{-1}$ because they are mutual inverse elements, thus we obtain

$$\text{(iii)} \quad (A_i * (A_m)^{-1}) * (A_m * A_k) = A_i * A_k. \tag{2.5}$$

Accordingly, using (2.5) and taking A_i and A_k as A_j and A_j^{-1}, respectively, we have

$$\text{(iv)} \quad (A_j * A_j^{-1}) * (A_j * A_j^{-1}) = A_j * A_j^{-1} \equiv I_{jL} * I_{jL} = I_{jL}. \tag{2.6}$$

In a general expression (2.5), taking A_i, A_m^{-1}, A_m and A_k as A_j, A_j, A_j^{-1} and A_j^{-1}, respectively, which use the property of mutual inverse elements of A_j and A_j^{-1}, we obtain

$$(A_j * A_j) * (A_j^{-1} * A_j^{-1}) = A_j * A_j^{-1} = I_{jL}. \tag{2.7}$$

Due to the arbitrary property of A_i and A_k in (2.5) and taking A_i as A_j, we deduce: $\forall A_j, A_j^{-1}, A_k, \exists A_{jk}$ so that

$$(A_j * A_j^{-1}) * (A_j * A_k) = A_j * A_k = I_{jL} * A_{jk} = A_{jk}, \tag{2.8}$$

where we have used (2.3), i.e., $A_j * A_k = A_{jk} = A_l$, then I_{jL} is a left identity, it satisfies a kind of binary operation of a new identity.

Because the above discussions form a general algebraic system, theorems (i) and (iii) satisfy QCP, and I_{jL} in the deduced (2.8) is the left identity, therefore, the algebraic system satisfying theorems (i) and (iii) is called as left causal algebra, and (2.6–2.8) are just the corollaries of theorems (i) and (iii).

3 Right Causal Algebra

Because left causal algebra has the left identity and the deduced corollaries from theorems (i) and (iv) of left causal algebra, due to the symmetrical properties between the left and right causal algebras, right causal algebra should have a right identity and the deduced corollaries from the theorems of right causal algebra. And the closure theorem (i) is a general basic theorem of different algebras, the symmetrical property demands that the closure theorem (i) is still a basic theorem of right causal algebra.

Analogous to the discussions about theorem (iii), due to the inverse symmetry of theorem (iii) of right causal algebra, we have axiom (iii′)

$$\text{(iii}'\text{)} \quad (A_i * A_m) * ((A_m)^{-1} * A_k) = A_i * A_k, \tag{3.1}$$

where the mark sign$(A_m)^{-1}$ is a general inverse element relative to A_m in S.

Accordingly, using (3.1), we deduce

$$\text{(iv}'\text{)} \quad (A_j^{-1} * A_j) * (A_j^{-1} * A_j) = A_j^{-1} * A_j \equiv I_{jR} * I_{jR} = I_{jR}. \tag{3.2}$$

In a general expression (3.1), taking A_i, A_m, A_m^{-1} and A_k as A_j^{-1}, A_j^{-1}, A_j and A_j, respectively, which use the property of mutual inverse elements of A_j and A_j^{-1}, we obtain

$$(A_j^{-1} * A_j^{-1}) * (A_j * A_j) = A_j^{-1} * A_j = I_{jR}, \tag{3.3}$$

which is a new operation relation.

Due to the arbitrary property of A_i and A_k in (3.1) and taking A_k as A_j and fetch $m = j$, we deduce: $\forall A_i, A_j, A_j^{-1}, A_m = A_{ij}$, there is the following relation

$$(A_i * A_j) * (A_j^{-1} * A_j) = A_i * A_j = A_{ij} * I_{jR} = A_{ij}, \text{ or } A_m * I_{jR} = A_m \tag{3.4}$$

where I_{jR} is a right identity, it satisfies a kind of binary operation of a new identity.

Because the above discussions form a general algebraic system, theorems (i) and (iii′) satisfy QCP, and I_{jR} in the deduced (3.4) is the right identity, therefore, the algebraic system satisfying theorems (i) and (iii′) is called as right causal algebra, and (3.2–3.4) are just the corollaries of theorems (i) and (iii′).

4 Left and Right Causal Decomposition Algebras

In left causal algebra, when the algebra satisfies cancellation law with general property, i.e., is decomposable, we have

$$(A_i * A_j^{-1}) * (A_j * A_k) = A_i * [A_j^{-1} * (A_j * A_k)] = A_i * A_k. \tag{4.1}$$

In fact, (4.1) is a cancellation condition of reciprocal eliminable axiom,then we may obtain the simplified reciprocal eliminable theorem

$$\text{(v)} \quad A_j^{-1} * (A_j * A_k) = A_k, \tag{4.2}$$

because using $A_i * [A_j^{-1} * (A_j * A_k)] = A_i * A_k$ one may generally deduce $A_i * ([A_j^{-1} * (A_j * A_k)] - A_k) = 0$. And because (v) is the solution under the conditions of existing cancellation law and making (2.5) decomposable, therefore, the left causal algebra is changed into a new algebra that is called as left causal decomposition algebra. Accordingly, left causal decomposition algebra consists of axioms (i), (iii) and (v), the other corollaries of left causal decomposition algebra can be deduced from axioms (i), (iii) and (v), e.g., multiplying (4.2) from the left by A_i, and using theorem (iii) we obtain the corollary (4.1), which mean that theorems (iii) and (v) are equivalent to theorem (iii) and cancellation law.

Because of the symmetrical properties between the left and right causal decomposition algebras, the mathematical structure of right causal decomposition algebra is similar to that of left causal decomposition algebra. And analogous to the discussions from left causal algebra to right causal algebra, for right causal decomposition algebra, the closure theorem (i) is still a basic theorem of right causal decomposition algebra.

Therefore, in right causal algebra, when right causal algebra satisfies cancellation law with general property, namely, right causal algebra is decomposable, we deduce

$$(A_i * A_j) * (A_j^{-1} * A_k) = [(A_i * A_j) * A_j^{-1}] * A_k = A_i * A_k. \tag{4.3}$$

Specially to explain: for (3.1) $D(S) = (A_i * A_j) * (A_j^{-1} * A_k)$ is a general case, for (4.3) $D(S) = DS = [(A_i * A_j) * A_j^{-1}] * A_k$ is a simplified case.

Actually, (4.3) is a cancellation condition of reciprocal eliminable theorem, then we achieve the simplified reciprocal eliminable theorem

$$(\mathrm{v}') \quad A_k = (A_k * A_j) * A_j^{-1}. \tag{4.4}$$

(v′) is a solution under the conditions of existing cancellation law and making (3.1) decomposable, the right causal algebra is thus changed into a new algebra that is called as right causal decomposition algebra. Therefore, right causal decomposition algebra makes of theorems (i), (iii′) and (v′), the other corollaries of right causal decomposition algebra can be derived from theorems (i), (iii′) and (v′), e.g., multiplying (4.4) from the right by A_m, and using theorem (iii′) we obtain the corollary (4.3), which mean that theorems (iii′) and (v′) are equivalent to theorem (iii′) and cancellation law.

5 Causal Algebra

Now we enlarge the left and right causal algebras to causal algebra.

When we combine left and right causal algebras into causal algebra, the new causal algebra consists of theorems (i), (iii), (iii′), which mean that the new algebra have the left and right symmetry, namely, causal algebra has nothing to do with the appearing position of the general inverse element A_j^{-1} in theorems (iii) and (iii′), accordingly, for causal algebra, we obtain

$$(A_i * A_j^{-1}) * (A_j * A_k) = A_i * A_k, \tag{5.1}$$

$$(A_i * A_j) * (A_j^{-1} * A_k) = A_i * A_k, \tag{5.2}$$

namely, (5.1) is equal to (5.2).

Using (5.1) and (5.2), we can prove $(A_j^{-1})^{-1} = A_j$, i.e., in causal algebra, A_j^{-1} is a concrete inverse element of A_j.

Proof: In (5.2), we take A_j as A_j^{-1}, then we obtain

$$(A_i * A_j^{-1}) * ((A_j^{-1})^{-1} * A_k) = A_i * A_k. \tag{5.3}$$

Comparing (5.3) with (5.1), the proposition is proved.

The applications and discussions of causal algebra will be given in Sect. 7.

In following, we generalize causal algebra to causal decomposition algebra.

6 Causal Decomposition Algebra

We further extend the left and right causal decomposition algebras to causal decomposition algebra.

When we combine left and right causal decomposition algebras into causal decomposition algebra, the new causal decomposition algebra consists of theorems (i), (iii), (iii′), (v), (v′), which mean that the new algebra has the left and right symmetry, namely, causal decomposition algebra has nothing to do with the appearing position of the general inverse element A_j^{-1} in theorems (iii) and (iii′), and the algebra has the decomposable property, accordingly, for causal decomposition algebra, we obtain

$$(A_i * A_j^{-1}) * (A_j * A_k) = A_i * [A_j^{-1} * (A_j * A_k)] = A_i * A_k, \tag{6.1}$$

$$(A_i * A_j) * (A_j^{-1} * A_k) = [(A_i * A_j) * A_j^{-1}] * A_k = A_i * A_k. \tag{6.2}$$

In fact, (6.1) or (6.2) is a cancellation condition of reciprocal eliminable theorem, then the reciprocal eliminable theorem is reduced into the simplified reciprocal eliminable theorem with left and right symmetry as follows

$$(\mathrm{v}'') \quad A_j^{-1} * (A_j * A_k) = A_k = (A_k * A_j) * A_j^{-1}. \tag{6.3}$$

(v″) are two solutions under the conditions of existing cancellation law and making (2.5) and (3.1) decomposable, the left and right causal decomposable algebra are thus combined into a new algebra that is called as causal decomposition algebra. Therefore, causal decomposition algebra is made of theorems (i), (iii), (iii′) (v) and (v′), the other corollaries of causal decomposition algebra can be deduced from theorems (i), (iii), (iii′) (v) and (v′), e.g., multiplying (4.2) and (4.4), respectively, from the left and the right by A_m, and using axioms (iii) and (iii′) we obtain the corollaries (4.1) and (4.3), which are equivalent to theorems (v) and (v′).

7 Relations Between Causal Decomposition Algebra and Group

Causal decomposition algebra naturally contains the structure of group.

Now prove: In terms of theorems (i), (v), and (iii′), associative law can be deduced.

Multiplying (2.3) with A_j^{-1} from the left, and using theorem (v), we obtain

$$A_k = A_j^{-1} * A_l. \tag{7.1}$$

Making (7.1) multiplied from the left by A_m, and utilizing (2.2) and theorem (iii′), we acquire an important formula as follows

$$A_m * A_k = A_i * A_l. \tag{7.2}$$

Substituting (2.2) and (2.3) into (7.2), we finally deduce associative law as follows

$$(\mathrm{vi}) \quad (A_i * A_j) * A_k = A_i * (A_j * A_k), \quad i, j, k \in Z \tag{7.3}$$

that is, using theorems (i), (v), and (iii′), one may deduce associative law, the proposition is proved.

Because theorems (i) and (v) belong to left causal decomposition algebra, and theorem (iii′) belongs to right causal algebra, the algebra of making left causal decomposition algebra and right causal algebra as subalgebras is causal decomposition algebra, thus causal decomposition algebra naturally contains associative law.

Now we prove that causal decomposition algebra has the structure of group.

We first show that in causal decomposition algebra, A_i^{-1} is a direct inverse element of A_i, then prove that $A_i * A_i^{-1}$ is identity in set S.

Using the deduced associative law, theorems (v) and (v′) may be rewritten as

$$(A_j^{-1} * A_j) * A_k = A_k, \tag{7.4}$$

$$A_k * (A_j * A_j^{-1}) = A_k. \tag{7.5}$$

It is seen that (7.4) and (7.5) have more direct inverse element representations than those of (2.4) or theorems (iii & iii′).

Using (7.5) and the deduced associative law we have

$$(A_i^{-1} * A_i) * (A_i * A_i^{-1}) = A_i^{-1} * A_i. \tag{7.6}$$

On the other hand, utilizing (7.4) and the deduced associative law, we can have

$$(A_i^{-1} * A_i) * (A_i * A_i^{-1}) = A_i * A_i^{-1}. \tag{7.7}$$

Accordingly, in causal decomposition algebra, we obtain

$$A_i^{-1} * A_i = A_i * A_i^{-1}, \quad \text{i.e., } I_{iR} = I_{iL} = I_i, \forall i \in Z. \tag{7.8}$$

Using (7.5) and (7.8), we can have

$$(A_i * A_i^{-1}) * (A_j * A_j^{-1}) = A_i * A_i^{-1} = I_i. \tag{7.9}$$

Utilizing (7.4) and (7.8), we have

$$(A_i * A_i^{-1}) * (A_j * A_j^{-1}) = A_j * A_j^{-1} = I_j. \tag{7.10}$$

Therefore, we obtain $A_i * A_i^{-1} = A_j * A_j^{-1}$, this means that $A_i * A_i^{-1}$ does not depend on A_i, we thus may deduce

$$A_i^{-1} * A_i = A_i * A_i^{-1} = A_j * A_j^{-1} = A_j^{-1} * A_j = I \in S. \tag{7.11}$$

Then (7.4) and (7.5) are rewritten as

$$I * A_i = A_i * I = A_i. \tag{7.12}$$

In terms of (7.11) and (7.12), A_i^{-1} is an inverse element of A_i, I is identity. Because there are theorems (i), (vi), (7.11) and (7.12), we achieve that, in this situation, S is a group. It is watched what causal decomposition algebra is equivalent to group. Accordingly, in terms of the above research, we discover that the symmetric properties are originated from QCP. On the other hand, in terms of group, one can not deduce causal algebra, but similar to the discussions of this article, we can deduce the causal decomposition algebra. Thus in this Section we can find that group has the structure of causal decomposition algebra, because there are the applications of group to a lot of aspects of physics, the causal decomposition algebra may have corresponding applications.

8 Application and Discussion

Typical characteristics of left (right) causal algebras are: satisfying the closure property and theorem (iii) (theorem (iii′)),the closure property of the algebra shows the transfer property of an operation acting on system, and the reciprocal eliminable condition of theorem (iii) (theorem (iii′)) means that the transfer once takes place, the relative inverse operation can be cancelled.

For example, consider A_j as a spin angle around a fixed axis,and A_j^{-1} is the relative inverse spin angle, A_i and A_k are also different spin angles around the same fixed axis, the operation "$*$" expresses that the corresponding spin angles may be added, then closure theorem and the reciprocal eliminable theorem keep to stand.

On the other hand, because causal decomposition algebra contains group structure, applications of all selective laws of group symmetries in various fundamental interactions can be done in terms of causal decomposition algebra, i.e., in terms of group theory, then we don't repeat here, these discussions can be found, e.g., in Ref. [20].

In physics of high energy, using the closure theorem in left (right) causal algebra, we can deduce a general unified expression of interactions of different particles. Because these reactions must satisfy QCP, there must be the beneath closure expression

$$Q_i * Q_j = Q_k. \tag{8.1}$$

Equation (8.1) shows that in the interaction of producing a particle by two particles, the quantity obtained from operation "*" of a particle's physical quantities with another particle's physical quantities equates to the physical quantity of the yielded particle, i.e., before and after the interaction, the physical quantities are constant. For example, in terms of expression (8.1), we have the unified expression of the physical quantities of the concrete interaction of two general particles' composition as follows

$$Q_{\pi^0} * Q_{\pi^+} = Q_{K^+}, \tag{8.2}$$

where Q_{π^0}, Q_{π^+} and Q_{K^+} are different physical quantities of π^0 meson, π^+ meson and K^+ meson, respectively, i.e., Q_{π^0}, Q_{π^+} and Q_{K^+} can be noted as A_i, A_j and A_m, respectively. Accordingly, these different physical quantities satisfy theorem (i) (closure), for instance, Q may be electric charge, baryon number, angular momentum, parity, G parity, momentum and energy etc.

In the reactions of containing supersymmetric particles, using the causal algebra we can deduce the unified identical equation relative to different conservative physical quantities, because the reactions of containing supersymmetric particles must satisfy the QCP, there then necessarily exist a unified and general identical equation (8.1) of the physical quantities satisfying the QCP, e.g., for the beneath reaction of containing supersymmetric particles

$$q + q_s \Leftrightarrow g_s, \tag{8.3}$$

where q, q_s and g_s are a quark, squark and gluino, respectively, and q, q_s's charges may take $\pm 2/3$ or $\pm 1/3$ unit charge, and the charges have the same absolute values but their signs are contrary. Using (8.1) we deduce the unified and general identical equation as follows

$$Q_q * Q_{q_s} = Q_{g_s}, \tag{8.4}$$

where Q_q, Q_{q_s} and Q_{g_s} are different physical quantities of q, q_s and g_s, respectively, i.e., Q_q, Q_{q_s} and Q_{g_s} can be noted as A_i, A_j and A_k, respectively, and they may be color charges.

For the other reactions of containing supersymmetric particles, we have the reaction representations and identical equations as follows

$$e^+ + e_s^- \Leftrightarrow \gamma_s \quad \text{and} \quad Q_{e^+} * Q_{e_s^-} = Q_{\gamma_s}, \tag{8.5}$$

$$q_{1s} + q_{2s} \Leftrightarrow g \quad \text{and} \quad Q_{q_{1s}} * Q_{q_{2s}} = Q_g, \tag{8.6}$$

$$e_s^+ + e_s^- \Leftrightarrow \gamma \quad \text{and} \quad Q_{e_s^+} * Q_{e_s^-} = Q_\gamma, \tag{8.7}$$

where e^+ is a antielectron (i.e., positron), e_s^- selectron, γ_s photino, q_{1s} and q_{2s} squarks, g gluon, e_s^+ antiselectron, γ photon, and q_{1s} and q_{2s} may have $\pm 2/3$ or $\pm 1/3$ unit charge, and the charges must have the same absolute values but their signs are contrary, baryonic numbers of q_{1s} and q_{2s} have also the same absolute values and the signs are reverse. When all conservative physical quantities satisfy the general conservative equation (8.1), then which insures that the reactions of containing supersymmetric particles can take place.

For reactions of representation (8.3) and the first representations in (8.5–8.7), using (8.1) and supersymmetric particles' R number, i.e., $R = 3B + L + 2S$, where B, L and S are baryonic number, lepton number and spin, respectively, we may find a P_R symmetry in any supersymmetric particles' reactions, namely, for any particle, we can define the particle's P_R symmetry quantity

$$P_R = (-1)^R = (-1)^{3B+L+2S}. \tag{8.8}$$

Then we get the supersymmetric $P_R = (-1)^R$ symmetry of multiplying property in the reactions of representation (8.3) and the first representations in (8.5–8.7), we now prove the supersymmetric invariance.

According to Ref. [21, 22], we can conclude: Except spin of supersymmetric particles of corresponding non-supersymmetric particles of spin zero is zero plus $1/2$, spins of all supersymmetric particles of corresponding non-supersymmetric particles are spins of non-supersymmetric particles minus $1/2$, and except for the spins and masses of the all particles, all non-supersymmetric particles and corresponding supersymmetric particles have the complete same properties.

Because the conservative physical quantities in the reactions of representation (8.3) and the first representations in (8.5–8.7) must satisfy the causal algebraic equation (8.1), the conservative physical quantities of mutual adding and multiplying properties in the reactions must satisfy (8.4) and the second representations in (8.5–8.7), respectively, for example, physical multiplying quantities of supersymmetric P_R of the particles of both sides in reactions of representation (8.3) and the first representations in (8.5–8.7) are invariant, respectively, e.g., considering the supersymmetric $P_R = (-1)^R$ symmetry in the reaction of the first representations in (8.5), baryonic number B, lepton number L and spin S of e^+ positron, e_s^- selectron and γ_s photino are e^+'s $(0, 0, 1/2)$, e_s^-'s $(0, 0, 0)$ and γ_s's $(0, 0, 1/2)$, then we deduce

$$P_R(e^+)P_R(e_s^-) = P_R(\gamma_s) = -1. \tag{8.9}$$

Namely, physical multiplying quantities of supersymmetric P_R of the particles of both sides in the reaction of the first representations in (8.5) are invariant. For the discussions of the other conservative physical quantities are analogous to the discussions, the detailed discussion about this aspect sees Ref. [20]. Equation (8.1) gives a unified expression that all physical quantities with multiplicative or additive property must satisfy, and for the all physical quantities, only when their all expressions are satisfied, the reactions can be determined to be observable reactions, (8.1) represents that the conversions of various physical quantities must maintain no-loss-no-gain satisfying QCP in whole process.

Furthermore, when existing l and m so that

$$Q_l * Q_m = Q_k, \tag{8.10}$$

then there is the reaction

$$Q_i * Q_j = Q_k = Q_l * Q_m, \tag{8.11}$$

where Q_k is a middle state, under the some circumstance this middle state can call resonance state, and (8.11) can be directly written as

$$Q_i * Q_j = Q_l * Q_m, \tag{8.12}$$

namely, (8.12) is a deduced identity in the interaction. For example, the collision of proton with π^- meson, there exists a relation of physical quantities satisfying QCP as follows

$$Q_p * Q_{\pi^-} = Q_n * Q_{\pi^0}, \tag{8.13}$$

where Q_p, Q_{π^-}, Q_n and Q_{π^0} are remarked as A_i, A_j, A_l and A_m, respectively, i.e., proton, π^- meson, neutron and π^0 meson.

Furthermore, using reciprocal eliminable theorem in left causal algebra, we can deduce a general identity of various physical quantities, e.g., the high energy particle reaction

$$Q_u * Q_d = (Q_u * Q_{e^-}) * (Q_{e^+} * Q_d), \tag{8.14}$$

when the energy of the collision is higher, there exists the beneath reaction

$$Q_p * Q_n = (Q_p * Q_{\pi^-}) * (Q_{\pi^+} * Q_n), \tag{8.15}$$

where Q_p, $Q_{e^-}(Q_{\pi^-})$, $Q_{e^+}(Q_{\pi^+})$ and Q_n are general physical quantities of proton, electron (π^- meson), positive electron (π^+ meson) and neutron, respectively, namely, they may be viewed as A_i, A_j^{-1}, A_j and A_m respectively, thus they satisfy (5.1), where Q_i can, respectively, be electric charge, lepton number, hadron number, angular momentum and momentum etc. Therefore, (8.14) and (8.15) give the identical equations that their unified quantum numbers must satisfy, and for all these quantum quantities, while their identical equations are all satisfied, one can conclude that the reaction can happen. Equation (8.15) represents that the collision of proton and neutron of high energy yields the other products but proton and neutron, and the other products must be a pair of positive and negative particles, the particles of the right side may be changed into the particles of the left side, that is, the sum of the quantum numbers of the particles in before and after the reactions, respectively, keep no-loss-no-gain conservation. Therefore, this is a kind of direct concrete representation of causal algebra, which is relative to high energy physics.

Especially, in the reactions of particles of high energy, there are a lot of reactions that are analogous to expression (8.13), there exist no identity, but there are special inverse elements, which make that expression (8.13) be not group, nor the ring, so that, up to now, there is no any algebra that can severely and directly describe them, however, causal algebra given in this paper is just the tool of severely, naturally and directly describing real reactions of particle physics. And there are a lot of applications to physics, for instance, in family scattering of particles of high energy, there are a lot of these kinds of reactions [20]. For instance, further taking Q_k in (8.10) as Q_m in (8.12), then we can get the several particles' reaction expression

$$Q_i * Q_j = Q_l * (Q_{i'} * Q_{m'}), \tag{8.16}$$

again taking Q_k in (8.10) as $Q_{i'}$ and $Q_{m'}$ in (8.15), respectively, then we can obtain many particles' reaction expression

$$Q_i * Q_j = Q_l * [(Q_{i''} * Q_{m''}) * (Q_{i'''} * Q_{m'''})]. \tag{8.17}$$

If further operating like this, we can get general reaction expressions of more particles.

Because one can not derive causal algebra in terms of group, but one can deduce causal decomposition algebra in terms of group, and using causal decomposition algebra this paper obtains group, then for group's applications to many aspects of physics, the causal decomposition algebra has corresponding applications.

As the set of conditions of satisfying theorems (i), (vi) and expressions (7.11) and (7.12) is called as group, and the system in the process of any serious operations must satisfy QCP with the no-loss-no-gain characteristic. And it is seen that theorems (i), (vi) and expressions (7.11) and (7.12) are deduced from QCP and they all satisfy QCP, accordingly, QCP naturally builds up the unified connections between the four conditions.

So about the application of the choice rulers of group to the variety of the physical quantities, the same can be processed by the causal decomposition algebra, i.e., one can use the group theory to study, then no longer repeat here, see Ref. [20].

In essential, a kind of algebra is just a kind of set of some elements satisfying some rules of logic and quantitative calculation [17], the algebras given in this paper just satisfy these conditions of forming algebras, thus our investigations are consistent with modern algebraic theory.

9 Summary and Conclusion

This paper gives both the unification theory of different causal algebras and its application to theoretical physics. That is, this paper educes the left and right causal algebras, left and right causal decomposition algebras, causal algebra and causal decomposition algebras in terms of the QCP, the theorems of the systems of these algebras satisfy QCP, therefore, the relative algebras are called as the corresponding causal algebras, the other all consequences are all the deductions of these corresponding theorems.

Because the general algebraic system of satisfying theorems (i) and (iii) (or (iii′)) in Sect. 2 (or 3) not only obeys QCP but also contains left (or right) identity I_{jL} (or I_{jR}), therefore, the causal algebraic system is called as left (or right) causal algebra. When the left (or right) causal algebra satisfies cancellation law with general property, i.e., is decomposable, we may obtain that the left (or right) causal algebra is changed into a new algebra that is called as left (or right) causal decomposition algebra. And the mathematical structure of right causal decomposition algebra is similar to the mathematical structure of left causal decomposition algebra. When combining left and right causal algebras into causal algebra, the new causal algebra consists of theorems (i), (iii), (iii′), then, the new algebra have the left and right symmetry. Specially, left (or right) causal algebra and left (or right) causal decomposition algebra all have the dependences of positions of inverse operators A_i^{-1} in these algebraic structures, the physical systems with these algebraic structures are very many in general physical systems, e.g., in string field theories there are many symmetric structures relative to inverse operators and the operator's positions [23, 24], specially these algebras can be used to investigate the quantum mechanics and the field theories that they are depended on noncommutative geometry as a useful tool [25, 26], and the discussion of this paper about the symmetric properties can be applied to Noether theorem [27], therefore, these algebras are useful in describing corresponding physics.

When combining left and right causal decomposition algebras into causal decomposition algebra, the new causal decomposition algebra consists of theorems (i), (iii), (iii′), (v), (v′), the new algebra, thus, has the left and right symmetry, i.e., causal decomposition algebra has nothing to do with the appearing position of the general inverse element A_j^{-1} in theorems (iii) and (iii′), and the algebra has the decomposable property. And in terms of theorems (i), (v), and (iii′), associative law is deduced, it is further proved that causal decomposition algebra naturally contains the structure of group. On the other hand, in terms of group, one cannot deduce causal algebra, group has only the structure of causal decomposition algebra. Because there are the applications of group to a lot of aspects of physics, the causal decomposition algebra may have corresponding applications.

Further the paper gives the applications of the algebraic systems in particle physics, especially in the reactions of containing supersymmetric particles. Using (8.1) (deduced from QCP) and supersymmetric particles' R number ($R = 3B + L + 2S$), we get the supersymmetric $P_R = (-1)^{3B+L+2S}$ invariance of multiplying property in the reactions of containing supersymmetric particles.

In the reactions of particles of high energy, there are a lot of reactions that are analogous to a general expression (8.12), concretely see (8.13), there exists no identity, but there are special inverse elements, which make that expression (8.12) be not group, nor the ring, so that, up to now, there is no any algebra that can severely and directly describe them, however, causal algebra given in this paper is just the tool of severely, naturally and directly describing real reactions of particle physics. And there may be a lot of applications to physics in family scattering of particles of high energy.

It is seen that theorems (i), (vi) and expressions (7.11) and (7.12) are deduced from QCP and they all satisfy QCP, accordingly, QCP naturally builds up the unified connections between the four conditions. Satisfying QCP is a basic point of departures of modern cosmology and the all other sciences. Because of according to QCP, this paper gives not only the causal algebra but also causal decomposition algebra, and only the causal decomposition algebra is equivalent to group, then the investigations relative to group can be generalized to the other all algebras except for the causal decomposition algebra, the new general algebraic systems can affect the researches, applications and development of ring, group and etc algebraic systems in high energy physics. Furthermore, the relative branch sciences and application investigations relative to group and the other symmetries in physics can be studied again by means of the deduced different algebraic systems. And QCP is useful for investigating many physical systems, e.g., see Refs. [28–31].

Acknowledgements The authors are grateful for Prof. Z.P. Li and Prof. A.Z. Yang for useful discussion and comment. The work is supported by the Project of Knowledge Innovation Program (PKIP) of Chinese Academy of Sciences (Grant No. KJCX2.YW.W10), National Natural Science Foundation of China (Grant No. 10875009) and Beijing Natural Science Foundation (Grant No. 1072005).

References

1. Pokorski, S.: Gauge Field Theories. Cambridge University Press, Cambridge (1987)
2. Huang, K.S.: Quarks, Leptons & Gauge Fields. World Scientific, Singapore (1982)
3. Ajdu, Z., Pokorski, S., Wroblewski, A.K.: Frontiers in Particle Physics: Proceedings of the XII Warsaw Symposium on Elementary Particle Physics. World Scientific, Singapore (1990)
4. Cornwell, J.A.: Group Theory in Physics, vol. I, II. Academic Press, London (1984)
5. Shifman, M., Turbiner, A.: Phys. Rev. A **59**, 1791 (1999)
6. Lebed, R.F., Suzuki, M.: Phys. Rev. D **44**, 829 (1991)
7. Chaichian, M., Ellinas, D., Kulish, P.: Phys. Rev. Lett. **65**, 980 (1990)
8. Anderson, J.T.: Phys. Rev. D **23**, 1856 (1981)
9. Huang, Y.C.: Mech. Res. Commun. **30**, 567 (2003)
10. Huang, Y.C., Lee, X.G., Shao, M.X.: Mod. Phys. Lett. A **21**, 1107 (2006)
11. Huang, Y.C., Lin, B.L.: Phys. Lett. A **299**, 644 (2002)
12. Huang, Y.C., Ma, F.C., Zhang, N.: Mod. Phys. Lett. B **18**, 1367 (2005)
13. Huang, Y.C., Weng, G.: Commun. Theor. Phys. **44**, 757 (2005)
14. Weyl, H.: Symmetry. Princeton University Press, Princeton (1952)

15. Nash, C., Sen, S.: Topology and Geometry for Physicists. Academic Press, San Diego (1983)
16. Chern, S.S.: Vector Bundles with a Connection, Studies in Global Differential Geometry. Mathematical Association of America, Providence (1988)
17. Burris, S., Sankappanavar, H.P.: A Course in Universal Algebra. Springer, Berlin (1981)
18. Yang, S.L.: Quantum dimensions of representations of Hopf algebras. Algebra Colloq. **5**(4), 459–464 (1998)
19. Xiao, J., Yang, S.L.: Algebras and Representation Theory **4**(5), 491–501 (2001)
20. Nachtmann, O.: Elementary Particle Physics—Concepts and Phenomena. Translated by A. Lahee and W. Wetzel. Springer, Berlin (1990)
21. Llewellyn Smith, C.H.: Supersymmetry and its experimental consequences, Phys. Rep., 24 (1982)
22. Ferrara, S.: Supersymmetry. Elsevier, Amsterdam (1987)
23. Polchinski, J.: String Theory, vol. I, II. Cambridge University Press, New York (1998)
24. Davies, P.C.W., Brown, J.: Superstrings. Cambridge University Press, Cambridge (1988)
25. Connes, A.: Noncommutative Geometry. Acad. Press, San Diego (1994)
26. Doplicher, S., Longo, R. (eds.): Noncommutative Geometry-Lectures. Given at the C.I.M.E. Summer School held in Martina Franca, Italy, September 3–9, 2000, vol. 1831
27. Li, Z.P.: Eur. Phys. J. C **27**, 866 (2003)
28. Huang, Y.C., Yu, C.X.: Phys. Rev. D **75**, 044011 (2007)
29. Liao, L., Huang, Y.C.: Ann. Phys. (N. Y.) **322**, 2469 (2007)
30. Huang, Y.C., Huo, Q.H.: Phys. Lett. B **662**, 290 (2008)
31. Huang, Y.C., Liao, L., Lee, X.G.: Eur. Phys. J. C **60**, 481 (2009)